

## Chapter Two - Energy Balance Without Chemical Reaction

- 22.1 The Concept of the Conservation of Energy
- 22.2 Energy Balances for Closed, Unsteady-State Systems
- 22.3 Energy Balances for Closed, Steady-State Systems
- 22.4 Energy Balances for Open, Unsteady-State Systems
- 22.5 Energy Balances for Open, Steady-State Systems

### 22.1 The Concept of the Conservation of Energy

As you know, the principle of the **conservation of energy** states that the total energy of the *system plus the surroundings* can neither be created nor destroyed.

### 22.2 Energy Balances for Closed, Unsteady-State Systems

Unsteady state means that the state of the material inside the system changes. You know that for a closed, unsteady-state system the macroscopic mass balance for a given time interval in words is

$$\left\{ \begin{array}{l} \text{accumulation of} \\ \text{mass within the} \\ \text{system boundary} \\ \text{from } t_1 \text{ to } t_2 \end{array} \right\} = \left\{ \begin{array}{l} \text{net transfer of mass into} \\ \text{the system through the} \\ \text{system boundary} \\ \text{from } t_1 \text{ to } t_2 \end{array} \right\} - \left\{ \begin{array}{l} \text{net transfer of mass out} \\ \text{of the system through} \\ \text{the system boundary} \\ \text{from } t_1 \text{ to } t_2 \end{array} \right\}$$

and in symbols is  $\Delta m_{\text{system}} = m_{\text{in}} - m_{\text{out}}$  (22.1)

In this book, rates have overlay dots on  $m$  ( $\dot{m}$ ), and specific (per unit mass) values will have overlay carets ( $\hat{m}$ ) placed on  $m$ .

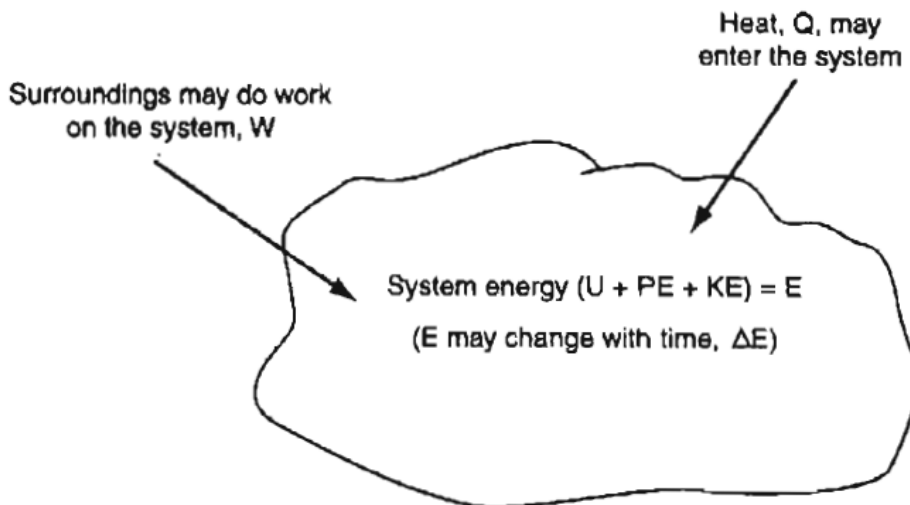
By analogy we can write the macroscopic **energy balances** for a closed, unsteady-state system for a given time interval in words as

$$\left\{ \begin{array}{l} \text{accumulation of} \\ \text{energy within} \\ \text{the system} \\ \text{from } t_1 \text{ to } t_2 \end{array} \right\} = \left\{ \begin{array}{l} \text{net transfer of energy} \\ \text{into the system through} \\ \text{system boundary} \\ \text{from } t_1 \text{ to } t_2 \end{array} \right\} - \left\{ \begin{array}{l} \text{net transfer of energy} \\ \text{out of the system through} \\ \text{system boundary} \\ \text{from } t_1 \text{ to } t_2 \end{array} \right\}$$

and replacing the words with the symbols we used in Chapter 21

$$\Delta (U + PE + KE)_{\text{inside}} \equiv \Delta E = Q + W \quad (22.2)$$

We do not put a  $\Delta$  representing the final minus initial states symbol before  $Q$  or  $W$  because they are not state variables. Remember that  $Q$  and  $W$  are both *positive* when transferred into the system, and  $E$  represents the sum of  $(U + KE + PE)$  associated with *mass in the system* itself, as shown in Figure 22.1. (Be careful; in some books  $W$  is defined as positive when done by the system.)



**Figure 22.1** A closed, unsteady-state system with heat ( $Q$ ) and work ( $W$ ).

Also keep in mind that each term in Equation (22.2) represents the respective *net cumulative* amount of energy over the time interval from  $t_1$  to  $t_2$ , not the respective energy per unit time, a rate, which would be denoted by an overlay dot.

In closed systems, except for falling stones and cannon balls, the values of  $\Delta PE$  and  $\Delta KE$  in  $\Delta E$  are usually negligible or zero, hence often you see  $\Delta U = Q + W$  used as the energy balance. You will find that closed systems occur far less frequently than the open systems that we will discuss in Sections 22.4 and 22.5.

If the sum of  $Q$  and  $W$  is positive,  $\Delta E$  increases; if negative,  $\Delta E$  decreases. Can you say that  $\Delta E_{\text{system}} = -\Delta E_{\text{surroundings}}$ ? Of course. Does  $W_{\text{system}} = -W_{\text{surroundings}}$ ? Not necessarily, as you will learn in Chapter 27. For example, in Figure 22.2c the electrical work done by the surroundings on the system degrades into heating of the system, not in expanding its boundaries. But you can say  $(Q + W)_{\text{system}} = -(Q + W)_{\text{surroundings}}$ .

Figure 22.2 illustrates three examples of applying Equation (22.2) to closed, unsteady-state systems. In Figure 22.2a, 10 kJ of heat is transferred through the fixed boundary (bottom) of a vessel with 2 kJ being transferred out at the top during the same time period. Thus,  $\Delta U$  increases by 8 kJ. In Figure 22.2b, a piston does 5 kJ of work on a gas whose internal energy increases by 5 kJ. In Figure 22.2c, the voltage difference between the system and surroundings forces a current into a system in which no heat transfer occurs because of the insulation on the system.

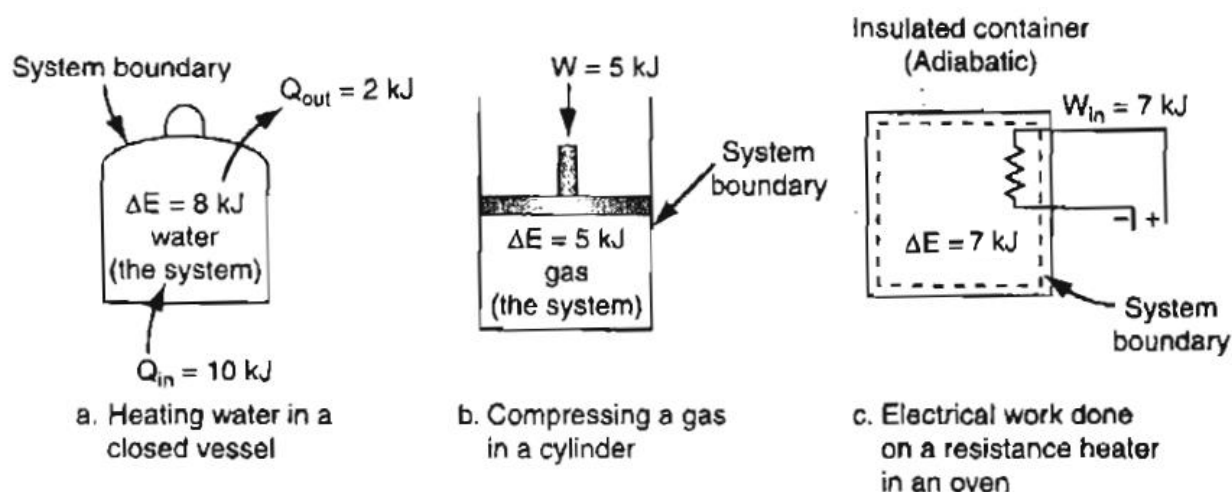
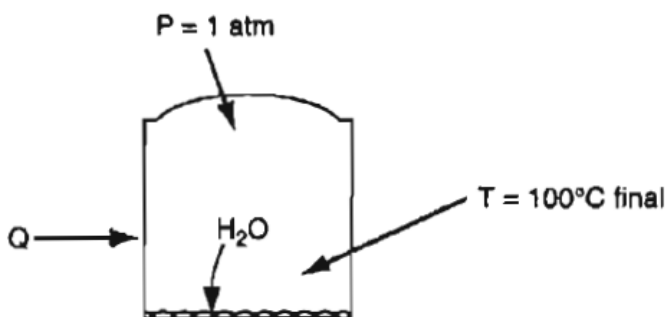


Figure 22.2 Examples of closed, unsteady-state systems that involve energy changes.

**EXAMPLE 22.1 Application of the Energy Balance to a Closed System**

Alkaloids are chemical compounds containing nitrogen that can be produced by plant cells. In an experiment, a closed vessel 1.673 m<sup>3</sup> in volume was filled with a dilute water solution containing two alkaloids, ajmalicine, and serpentine. The temperature of the solution was 10°C. To obtain an essentially dry residue of alkaloids, all of the water in the vessel (1 kg) was vaporized. Assume that the properties of water can be used as a substitute for the properties of the solution. How much heat has to be transferred to the vessel if 1 kg of saturated liquid water initially at 10°C is completely vaporized to the final conditions of 100°C and 1 atm? See Figure E22.1. Note the internal energy of water at 10 °C is 35 (kJ/kg) and at 100 °C is 2506 (kJ/kg).

**Figure E22.1****Solution**

Sufficient data is given in the problem statement to fix the initial state and the final state of the water. You can look up the properties of water in the steam tables or on the CD in the back of this book. Note that the specific volume of steam at 100°C and 1 atm is 1.673 m<sup>3</sup>/kg.

Initial state (liquid)		Final state (gas)
$p$	1 atm	1 atm
$T$	10.0°C	100°C
$\hat{U}$	35 kJ/kg	2506.0 kJ/kg

You can look up additional properties of water such as  $\hat{V}$  and  $\hat{H}$ , but they are not needed for the problem.

The system is closed, unsteady state so that Equation (22.2) applies

$$\Delta E = \Delta U + \Delta PE + \Delta KE = Q + W$$

Because the system (the water) is at rest,  $\Delta KE = 0$ . Because the center of mass of the water changes so very slightly,  $\Delta PE = 0$ . No work is involved (fixed tank boundary). You can conclude using a basis of

Basis: 1 kg H<sub>2</sub>O evaporated

$$Q = \Delta U = m\Delta\hat{U} = m(\hat{U}_2 - \hat{U}_1)$$

$$Q = \frac{1 \text{ kg H}_2\text{O}}{\text{kg}} \left| \frac{(2506.0 - 35) \text{ kJ}}{\text{kg}} \right| = 2471 \text{ kJ}$$

### EXAMPLE 22.2 Calculation of $\Delta U$ Using American Engineering Units

Saturated liquid water is cooled from 80°F to 40°F still saturated. What are  $\Delta\hat{E}$ ,  $\Delta\hat{U}$ ,  $\Delta\hat{H}$ ,  $\Delta\hat{p}$ , and  $\Delta\hat{V}$ ?

#### Solution

The system is closed and unsteady state, and thus analogous to Example 22.1. The properties listed below are from the steam tables for saturated liquid (at its vapor pressure).

	Initial Conditions	Final Conditions
$p^*$ (psia)	0.5067	0.1217
$\hat{V}$ (ft <sup>3</sup> /lb)	0.01607	0.01602
$\hat{H}$ (Btu/lb)	48.02	8.05

Basis: 1 lb water

Then

$$\Delta\hat{p} = (0.1217 - 0.5067) = -0.385 \text{ psia}$$

$$\Delta\hat{V} = (0.01602 - 0.01607) = \text{negligible value}$$

$$\Delta\hat{H} = (8.05 - 48.02) = -39.97 \text{ Btu/lb}$$

$$\hat{W} = 0$$

$$\Delta \widehat{KE} = 0$$

$$\Delta \widehat{PE} = 0$$

Now  $\Delta \hat{U} = \Delta \hat{H} - \Delta(p\hat{V})$

$$\Delta(p\hat{V}) = p_2\hat{V}_2 - p_1\hat{V}_1$$

$$\begin{aligned} &= \frac{0.1217 \text{ lb}_f}{\text{in.}^2} \left| \frac{144 \text{ in.}^2}{1 \text{ ft}^2} \right| \frac{0.01602 \text{ ft}^3}{\text{lb}_m} \left| \frac{1 \text{ Btu}}{778(\text{ft})(\text{lb}_f)} \right| \\ &\quad - \frac{0.5067 \text{ lb}_f}{\text{in.}^2} \left| \frac{144 \text{ in.}^2}{1 \text{ ft}^2} \right| \frac{0.01607 \text{ ft}^3}{\text{lb}_m} \left| \frac{1 \text{ Btu}}{778(\text{ft})(\text{lb}_f)} \right| \\ &= -1.141 \times 10^{-3} \text{ Btu/lb} \quad \text{a negligible quantity} \end{aligned}$$

Thus

$$\Delta \hat{U} = \Delta \hat{H} = -39.97 \text{ Btu/lb}$$

For a change in state of a liquid to a vapor,  $\Delta(p\hat{V})$  may not be negligible.

## 22.3 Energy Balances for Closed, Steady-State Systems

Recall that steady state means the accumulation in the system is zero, and that the flows of  $Q$  and  $W$  in and out of the system are constant. They can actually vary in the process, of course, but we are really interested only in their net cumulative values over a time interval, and look at only the final and initial conditions for  $\Delta E$ .

How should Equation (22.2) be modified to analyze steady-state systems? All you have to do is realize that inside the system

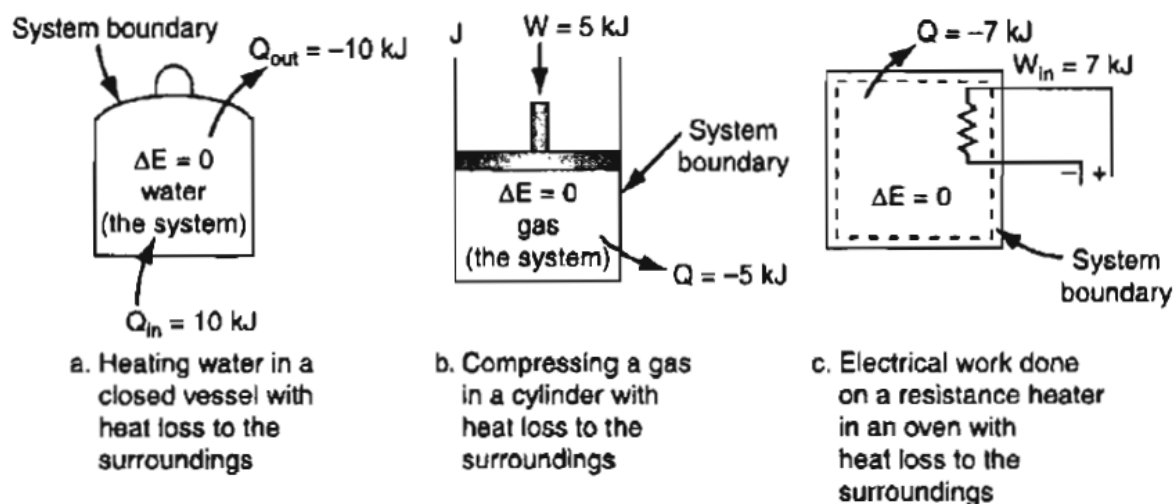
$$\begin{aligned} \Delta KE &= 0 & \Delta U &= 0 \\ \Delta PE &= 0 & \Delta E &= 0 \end{aligned}$$

hence  $Q + W = 0$  (22.4)

If you rearrange Equation (22.4), you get  $W = -Q$ , meaning that all of the work done on a closed, steady-state system must be transferred out as heat ( $-Q$ ). However, ironically, the reverse is false, namely the heat added to a *closed, steady-state system*  $Q$  does not always equal the work done by the system ( $-W$ ).

Figure 22.3 illustrates some examples of closed, steady-state systems with energy interchange. Contrast them with the figures in Figure 22.2.

In the processes shown in Figure 22.3, you can calculate  $Q$  and  $W$  as follows:



**Figure 22.3** Examples of closed, steady-state systems that involve energy changes.

- Fig. a.  $W = 0$  and hence  $Q = 0$  ( $Q$  is the cumulative *net* heat transfer in Equation (22.4).)
- Fig. b.  $W = 5 \text{ kJ}$  and hence  $Q = -5 \text{ kJ}$
- Fig. c.  $W = 7 \text{ kJ}$  and hence  $Q = -7 \text{ kJ}$

In summary, for a closed, steady-state system, the energy balance reduces to Equation (22.4).

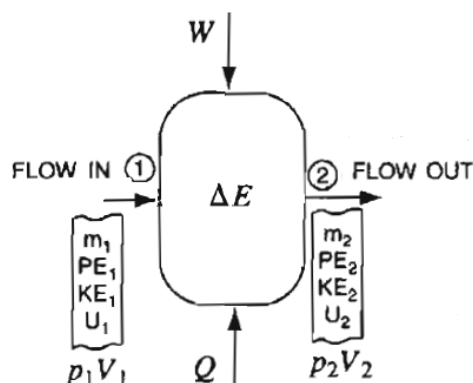
## 22.4 Energy Balances for Open, Unsteady-State Systems

Now that we have discussed closed systems, it is time to focus on processes represented by open systems, the occurrence of which is much more common than closed systems.

In an open, unsteady-state system, the accumulation term ( $\Delta E$ ) in the energy balance can be nonzero because

1. the mass in the system changes,
2. the energy per unit mass in the system changes, and
3. both 1 and 2 occur.

You know that when mass flows in and out of a system the mass carries energy along with it. What types of energy? Just the same types that are associated with the mass inside the system, namely  $U$ ,  $PE$ , and  $KE$ . All you have to do, then, is add these three types of energy associated with each stream going in and out of the system to the energy transfers of  $Q$  and  $W$  in the energy balance, Equation (22.2), to



**Figure 22.4** An open, unsteady-state system. The system is inside the boundary. (1) and (2) denote sections for entering and exiting mass flows, respectively.

make a **general energy balance** that is valid for both open systems and closed systems without reaction.  $\Delta E$  will still correspond the *final state within the system minus the initial state within the system*.

Figure 22.4 illustrates a general open system with one stream entering and one leaving. We are not concerned with the details of the process—only with the energy transfers into and out of the system, and changes within the system as a whole. Table 22.1 lists the notation that we will use in formulating the general energy balance. The overlay caret (^) still means the energy term per unit mass, and  $m$  with the subscript  $t_1$  or  $t_2$  denotes mass at time  $t_1$  or  $t_2$ , respectively, whereas  $m$  with the subscript 1 or 2 denotes the total flow of mass between  $t_1$  and  $t_2$  at section 1 or 2, respectively, in Figure 22.4.

Note that in Table 22.1 we have split the work term into parts, as mentioned in Chapter 21.



For a specified time interval, you can assemble each of the terms that will be in the general energy balance using the symbols given in Table 22.1 as follows:

**Accumulation in the system from  $t_1$  to  $t_2$ :**

$$\Delta E \equiv m_{t_2} (\hat{U} + \widehat{KE} + \widehat{PE})_{t_2} - m_{t_1} (\hat{U} + \widehat{KE} + \widehat{PE})_{t_1}$$

**Energy transfer in with mass from  $t_1$  to  $t_2$ :**  $(\hat{U}_1 + \widehat{KE}_1 + \widehat{PE}_1)m_1$

**Energy transfer out with mass from  $t_1$  to  $t_2$ :**  $(\hat{U}_2 + \widehat{KE}_2 + \widehat{PE}_2)m_2$

**Net energy transfer by heat transfer in and out from  $t_1$  to  $t_2$ :**  $Q$

**Net energy transfer by shaft, mechanical, or electrical work in and out from  $t_1$  to  $t_2$ :**  $W$

**Net energy transfer by work to introduce and remove mass from  $t_1$  to  $t_2$ :**

$$p_1 \hat{V}_1 m_1 - p_2 \hat{V}_2 m_2$$

**TABLE 22.1 Summary of the Symbols to be Used in the General Energy Balances**

Accumulation term (inside the system)		
Type of energy in the system	At time $t_1$	At time $t_2$
Internal	$U_{t_1}$	$U_{t_2}$
Kinetic	$KE_{t_1}$	$KE_{t_2}$
Potential	$PE_{t_1}$	$PE_{t_2}$
Mass of the system	$m_{t_1}$	$m_{t_2}$
Energy accompanying mass transport (through the system boundary) during the time interval $t_1$ to $t_2$		
Type of energy	Transport in	Transport out
Internal	$U_1$	$U_2$
Kinetic	$KE_1$	$KE_2$
Potential	$PE_1$	$PE_2$
Mass of the flow	$m_1$	$m_2$
Net heat exchange between the system and the surroundings during the interval $t_1$ to $t_2$	$Q$	

Work terms (exchange with the surroundings) during the interval  $t_1$  to  $t_2$

Net shaft, mechanical, and electrical work

$$W \begin{cases} W_{\text{shaft}} \\ W_{\text{mechanical}} \\ W_{\text{electrical}} \end{cases}$$

Flow work done on the system to introduce material into the system

$$m_1(p_1\hat{V}_1)$$

Flow work done on the surroundings to remove material from the system

$$-m_2(p_2\hat{V}_2)$$

The quantities  $p_1\hat{V}_1$  and  $p_2\hat{V}_2$  probably need a little explanation. They represent the so-called “ $pV$  work,” “pressure energy,” “flow work,” or “flow energy,” that is, the work done by the surroundings to put a mass of matter into the system at boundary 1 in Figure 22.4, and the work done by the system on the surroundings as a unit mass leaves the system at boundary 2, respectively. Because the pressures at the entrance and exit to the system are deemed to be constant for differential displacements of mass, the work done per unit mass by the surroundings on the system adds energy to the system at boundary 1:

$$\hat{W}_1 = \int_0^{\hat{V}_1} p_1 d\hat{V} = p_1(\hat{V}_1 - 0) = p_1\hat{V}_1$$

where  $\hat{V}$  is the volume per unit mass. Similarly, the work done by the fluid on the surroundings as the fluid leaves the system is  $W_2 = -p_2\hat{V}_2$ .

If we now combine all of the terms listed above into an energy balance, we get a somewhat formidable equation

$$\Delta E = (\hat{U}_1 + \hat{K}E_1 + \hat{P}E_1)m_1 - (\hat{U}_2 + \hat{K}E_2 + \hat{P}E_2)m_2 + Q + W + p_1\hat{V}_1m_1 - p_2\hat{V}_2m_2 \quad (22.5)$$

To simplify the notation in Equation (22.5), let us add

$$p_1\hat{V}_1m_1 \text{ to } \hat{U}_1m_1 \quad \text{and} \quad p_2\hat{V}_2m_2 \text{ to } \hat{U}_2m_2$$

to get

$$\Delta E = [(\hat{U}_1 + p_1\hat{V}_1) + \hat{K}E_1 + \hat{P}E_1]m_1 - [(\hat{U}_2 + p_2\hat{V}_2) + \hat{K}E_2 + \hat{P}E_2]m_2 + Q + W \quad (22.5a)$$

Next, introduce  $\hat{H}$  via the expression  $\hat{U} + p\hat{V} = \hat{H}$  into Equation (22.5a) to get

$$\Delta E = (\hat{H}_1 + \hat{K}E_1 + \hat{P}E_1)m_1 - (\hat{H}_2 + \hat{K}E_2 + \hat{P}E_2)m_2 + Q + W \quad (22.5b)$$

You can now see why the variable called enthalpy appears in the general energy balance.

Because Equation (22.5b) is so long, we have a suggestion. To help you memorize the energy balance, change the notation a bit to what is used in many texts:

$$\Delta E = E_{t_2} - E_{t_1} = Q + W - \Delta(H + KE + PE) \quad (22.6)$$

where

$$E_t = (U + KE + PE)_t \text{ refers to inside the system at time } t$$

In Equation (22.6) the delta symbol ( $\Delta$ ) **standing for a difference** then has two different meanings.

- In  $\Delta E$ ,  $\Delta$  means final minus initial in *time*.
- In  $\Delta(H + PE + KE)$ ,  $\Delta$  means *out of the system minus into the system*.

### EXAMPLE 22.3 Use of the General Energy Balance to Analyze an Open, Unsteady-State System

A rigid, well-insulated tank is connected to two valves. One valve goes to a steam line that has steam at 1000 kPa and 600K, and the other to a vacuum pump. Both valves are initially closed. Then the valve to the vacuum pump is opened, the tank is evacuated, and the valve closed. Next the valve to the steam line is opened so that the steam enters the evacuated tank very slowly until the pressure in the tank equals the pressure in the steam line. Calculate the final temperature of the steam in the tank.

#### Solution

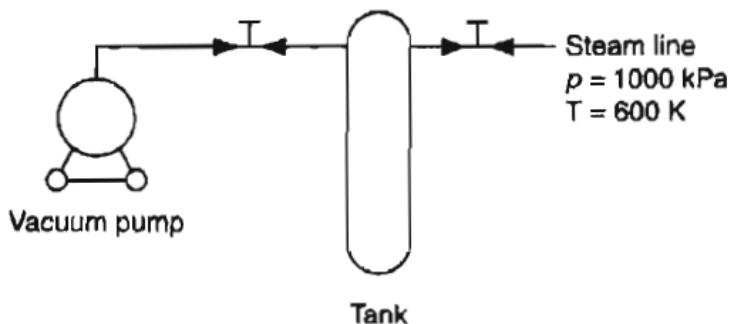


Figure E22.3a shows the process.

First, pick the tank as the system. If you do, the system is unsteady state (the mass increases in the system) and open. Pick a basis of 1 kg.

Next, get the data for steam at 1000 kPa and 600 K from the database in the CD in the back of this book: or can get it from steam table

$$\hat{U} = 2837.73 \text{ kJ/kg}$$

$$\hat{H} = 3109.44 \text{ kJ/kg}$$

$$\hat{V} = 0.271 \text{ m}^3/\text{kg}$$

Next, write down the general energy balance, Equation (22.6).

$$E_{t_2} - E_{t_1} = Q + W - \Delta(H + KE + PE) \quad (a)$$

and begin simplifying it. You can make the following assumptions:

1. No change occurs within the system for the  $PE$  and  $KE$ , hence  $\Delta E = \Delta U$ .
2. No work is done on or by the system because the tank is rigid; hence  $W = 0$ .
3. No heat is transferred to or from the system because it is well insulated, hence  $Q = 0$ .
4. The  $\Delta KE$  and  $\Delta PE$  of the steam entering the system are zero.
5. No stream exits the system, hence  $H_{out} = 0$ .
6. Initially no mass exists in the system, hence  $U_{t_1} = 0$ .

Consequently Equation (a) reduces to

$$U_{t_2} - 0 = -(H_{out} - H_{in})$$

$$\text{or} \quad \Delta U = U_{t_2} = m_{in} \hat{U}_{t_2} = H_{in} = m_{in} \hat{H}_{in} \quad (b)$$

To fix the final temperature of the steam in the tank, you have to determine two properties of the steam in the tank—any two. One value is given:  $p = 1000$  kPa. What other property is known? Not  $T$  nor  $\hat{V}$ . But you can use Equation (b) to calculate  $\hat{U}_{t_2}$  because

$$\hat{U}_{t_2} = \hat{H}_{in} = 3109.44 \text{ kJ/kg} \quad (c)$$

and find from interpolating in the steam tables at  $p = 1000$  kPa that  $T = 764$  K.

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If there is more than *one input and output stream* for the system, you will find it becomes convenient to calculate the properties of each stream separately and sum the respective inputs and outputs so that Equation (22.5b) becomes the **general energy balance** (without reaction occurring)

$$\Delta E = E_{t_2} - E_{t_1} = \sum_{\substack{\text{input} \\ \text{streams} \\ i=1}}^M m_i(\hat{H}_i + \widehat{KE}_i + \widehat{PE}_i) - \sum_{\substack{\text{output} \\ \text{streams} \\ o=1}}^N m_o(\hat{H}_o + \widehat{KE}_o + \widehat{PE}_o) + Q + W \quad (22.7)$$

where  $E_t = U_t + KE_t + PE_t$  inside the system

$M$  = number of input streams

$N$  = number of output streams

$i$  = input stream

$o$  = output stream

Equation (22.7) applies to open and closed systems as well as unsteady-state and steady-state systems.

## 22.5 Energy Balance for Open, Steady-State Systems

As we mentioned earlier, you will find that the preponderance of industrial processes operate under continuous, steady-state conditions. Most processes in the refining and chemical industries are open, steady-state systems. You will find that continuous processes are most cost effective in producing high-volume products.

Because steady state means that all of the state properties ( $T$ ,  $p$ , etc.) and the mass within the system are invariant with respect to time, the final and initial states of the system are the same, and  $\Delta E = 0$ . Continuous means the flows of heat and mass into and out of the system are constant (even though they are not, they often can be hypothetically assumed to be some average flows). Consequently, Equation (22.6) becomes with  $\Delta E = 0$

$$Q + W = \Delta(H + PE + KE) \quad (22.8)$$

When are  $\Delta PE$  and  $\Delta KE$  negligible? Because the energy terms in the energy balance in most open processes are dominated by  $Q$ ,  $W$ , and  $\Delta H$ ,  $\Delta PE$ , and  $\Delta KE$  only infrequently need to be used in Equation (22.8).

As a result, the equation most commonly applied to open, steady-state processes does not include any potential and kinetic energy changes

$$Q + W = \Delta H \quad (22.9)$$

### Example 22.4 Application of the Energy Balance to a Open, Steady-State System, a Heat Exchanger

Milk (essentially water) is heated from 15°C to 25°C by hot water that goes from 70°C to 35°C, as shown in Figure E22.4. What assumptions can you make to simplify Equation (22.8), and what is the rate of water flow in kg/min per kg/min of milk?

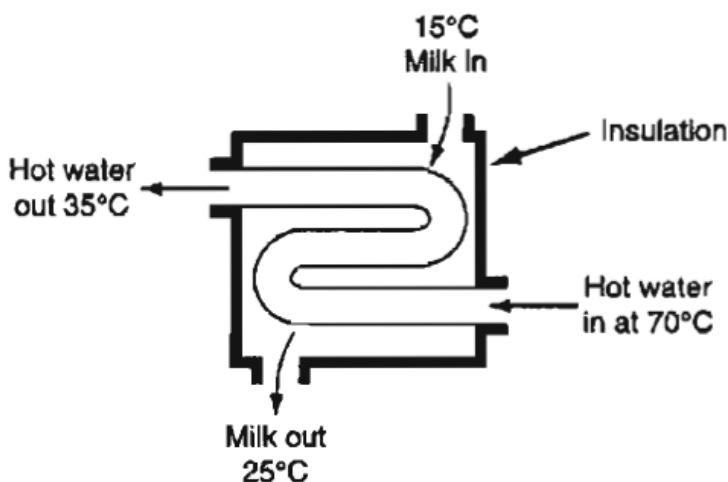


Figure E22.4

### Solution

Pick the milk plus the water in the tank as the system. You could pick the milk (or the water) as the system so that heat is transferred from the water to the milk (the two fluids do not mix), but combining the two fluids as the system makes the analysis simple. What assumptions can be made to simplify Equation (22.8)? Here they are:

1. Certainly  $\Delta KE$  and  $\Delta PE$  are zero.
2.  $Q = 0$  because of the way we picked the system—it is insulated.
3.  $W = 0$ .

With these assumptions Equation (22.8) becomes

$$\Delta H = 0 ! \quad (a)$$

Next, we collect the properties of water from the database or from the steam tables. Assume that the water is saturated, and that the milk has the same properties as water.

$T(^{\circ}\text{C})$	$\Delta\hat{H}(\text{kJ/kg})$
15	62.01
25	103.86
35	146.69
70	293.10

Let the basis be 1 min (the same as 1 kg of milk).

$$\frac{H_{\text{out}}}{[(1)\hat{H}_{\text{milk}, 25^{\circ}\text{C}} + (m)\hat{H}_{\text{water}, 35^{\circ}\text{C}}]} - \frac{H_{\text{In}}}{[(1)\hat{H}_{\text{milk}, 15^{\circ}\text{C}} + (m)\hat{H}_{\text{water}, 70^{\circ}\text{C}}]} = 0 \quad (\text{b})$$

$$[103.86 + (m)146.69] - [62.01 + (m)293.10] = 0$$

$$m = \frac{41.85}{146.41} = (0.29 \text{ kg hot water/min})/(\text{kg milk/min})$$

Let's repeat the problem solution but this time pick the milk as the system. The surroundings are comprised of the water and the tank. Now the assumptions to make in connection with using Equation (22.8) are:

1.  $\Delta KE = \Delta PE = 0$
2.  $W = 0$

Then Equation (22.8) reduces to

$$Q_{\text{water} \rightarrow \text{milk}} = \Delta H_{\text{milk}} = (1 \text{ kg})(\Delta\hat{H}_{\text{milk}} \text{ kJ/kg}) \quad (\text{c})$$

We know  $\Delta H$  of the milk. What is  $Q$ ?  $Q$  has to be calculated from the properties of the water. Equation (22.8) gives for the water

$$-Q_{\text{water} \rightarrow \text{milk}} = (m \text{ kg})(\Delta\hat{H}_{\text{water}} \text{ kJ/kg}) \quad (\text{d})$$

The minus sign appears before  $Q$  because the heat transfer is out of the water into the milk.

If you combine Equations (c) and (d), you get

$$(1)\Delta H_{\text{milk}} - (m)\Delta\hat{H}_{\text{water}} = 0 \quad (\text{e})$$

or

$$((1)\hat{H}_{\text{milk}, 25^{\circ}\text{C}} - (1)\hat{H}_{\text{milk}, 15^{\circ}\text{C}}) - (m\Delta\hat{H}_{\text{water}, 70^{\circ}\text{C}} - m\Delta\hat{H}_{\text{water}, 35^{\circ}\text{C}}) = 0 \quad (\text{f})$$

Rearrange Equation (f) to get

$$(1)(\Delta\hat{H}_{\text{milk}, 25^{\circ}\text{C}} + m\Delta\hat{H}_{\text{water}, 35^{\circ}\text{C}}) - (\Delta\hat{H}_{\text{milk}, 15^{\circ}\text{C}} + m\Delta\hat{H}_{\text{water}, 70^{\circ}\text{C}}) = 0 \quad (\text{g})$$

Equation (g) is exactly the same as Equation (b), as you might anticipate. Just a different perspective was used in picking the system for analysis.

### EXAMPLE 22.5 Calculation of the Power Needed to Pump Water in an Open, Steady-State System

Water is pumped from a well (Figure E22.5) in which the water level is a constant 20 feet below the ground level. The water is discharged into a level pipe that is 5 feet above the ground at a rate of  $0.50 \text{ ft}^3/\text{s}$ . Assume that negligible heat transfer occurs from the water during its flow. Calculate the electric power required by the pump if it is 100% efficient and you can neglect friction in the pipe and the pump.

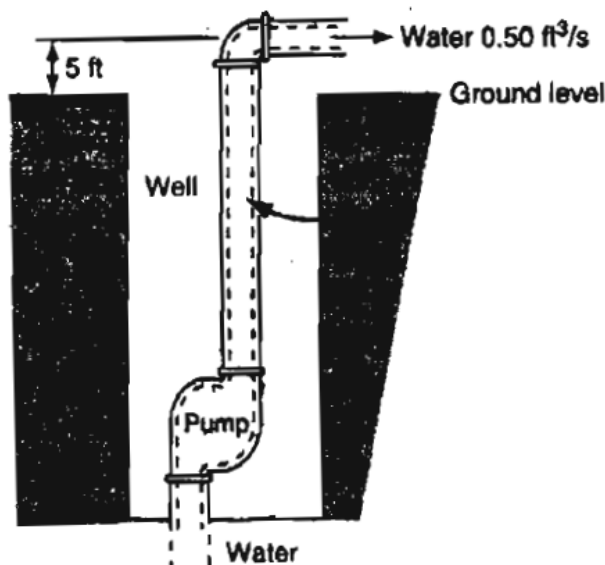


Figure E22.5

### Solution

We will pick as the system the pipe from the water level in the well to the place where the water that exits is at 5 feet above the ground including the pump. Some assumptions that will help in simplifying Equation (22.8) are:



1.  $Q = 0$  (given assumption)
2.  $\Delta KE \cong 0$  (negligible change in  $KE$ ; you can verify this statement if you have doubts—assume a linear velocity at the top of the pipe equal to 7 ft/s, which is a typical value found in industrial applications).

What about  $\Delta H$ ? Let us assume that the temperature of the water in the well is the same as the temperature of the water as it is discharged—a good assumption. Then Equation (22.8) reduces to

$$W = \Delta PE = mg(h_{\text{out}} - h_{\text{in}}) \quad (\text{a})$$

Choose a basis of 1 second. The mass flow is (say at 50°F)

$$\frac{0.50 \text{ ft}^3}{\text{s}} \left| \frac{62.4 \text{ lb}_m}{\text{ft}^3} \right| = 31.3 \text{ lb}_m \text{ water/s}$$

$$\begin{aligned} W = PE_{\text{out}} - PE_{\text{in}} &= \frac{31.3 \text{ lb}_m \text{ H}_2\text{O}}{\text{s}} \left| \frac{32.2 \text{ ft}}{\text{s}^2} \right| \left| \frac{25 \text{ ft}}{\text{s}^2} \right| \left| \frac{(\text{s}^2)(\text{lb}_f)}{32.2 (\text{ft})(\text{lb}_m)} \right| \left| \frac{1.055 (\text{kW})(\text{s}^2)}{778.2 (\text{lb}_f)(\text{ft})} \right| \\ &= 1.06 \text{ kW (1.42 hp)} \end{aligned}$$

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**Notice This**

**Home Work:** *Basic Principles and Calculation in Chemical Engineering*, 7<sup>th</sup> edition, **Problems of chapter 22, page 674 – 680.**